Topology

M. Math. I

Back Paper Examination

Instructions: All questions carry equal marks.

1. Let $\{A_\alpha\}$ be a collection of subsets of a topological space X such that for every $x \in X$, there exists an open set U_x containing x which intersects only finitely many A_α's. Then prove that

$$\overline{\cup A_\alpha} = \cup \overline{A_\alpha}$$

   Give an example of a topological space Y and subsets $\{B_\alpha\}$ for which this equality does not hold.

2. Let X be a topological space and $A \subset X$. Let $f : A \to Y$ be continuous with Y a Hausdorff space. Show that if f may be extended to a continuous function $g : \overline{A} \to Y$, then g is uniquely determined by f.

3. Give an example of a connected topological space which has infinitely many path components. Justify your answer.

4. Define a closed map between topological spaces. Let X be a compact space and Y be a Hausdorff space. Show that any continuous map $f : X \to Y$ is a closed map.

5. State Urysohn Lemma. Prove that a connected normal space consists of either a single point or an uncountable number of points.

6. Define a covering map. Let S^1 denote the set of complex numbers with modulus one. Prove that the map $z \rightarrowtail z^n$ is a covering map from S^1 to itself.